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CONTACT PROBLEM FOR TWO MICROINHOMOGENEOUS HALF-SPACES

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The interaction between two microinhomogeneous half-spaces in a macrohomogeneous stress-strain state is considered. The perturbation method is used to solve the problem. Finite expressions for the correlation functions and dispersion of deformations are obtained in the first order approximation. Numerical values of the coefficients characterizing the stress concentration at the boundary of interaction between the half-spaces are derived. The dependence of these coefficients on the ratio of elastic moduli of the media filling the half-spaces is studied. A particular solution of this problem for the case when the elastic moduli depend on two coordinates only, was given in [1].

1. Let the randomly inhomogeneous half-spaces $x_3 \geqslant 0$ and $x_3 \leqslant 0$ in which a macrohomogeneous stress-strain state prevails, interact along the plane $x_3 = 0$. We write the displacements, deformations and stresses in the form [2]

$$u_i^{(k)}(\mathbf{x}) = \langle u_i^{(k)}(\mathbf{x})\rangle + v_i^{(k)}(\mathbf{x}) \tag{1.1}$$
$$e_{ij}^{(k)}(\mathbf{x}) = \langle e_{ij}^{(k)}(\mathbf{x})\rangle + \varepsilon_{ij}^{(k)}(\mathbf{x})$$
$$\sigma_{ij}^{(k)}(\mathbf{x}) = \langle \sigma_{ij}^{(k)}(\mathbf{x})\rangle + \tau_{ij}^{(k)}(\mathbf{x})$$
$$(i, j = 1, 2, 3;\ k = 1, 2)$$

Here $v_i^{(k)}$, $\varepsilon_{ij}^{(k)}$, $\tau_{ij}^{(k)}$ denote the fluctuations in the values of the displacements, deformations and stresses about their mean values $\langle u_i^{(k)}\rangle$, $\langle e_{ij}^{(k)}\rangle$, $\langle \sigma_{ij}^{(k)}\rangle$ describing the macroscopically homogeneous stress-strain state. The index k assumes the value of 1 for the half-space $x_3 \geqslant 0$, and the value of 2 for $x_3 \leqslant 0$. The equations of equilibrium in the first order approximation have the form [3, 4]

$$\varkappa_k v_{i,ij}^{(k)} + v_{j,mm}^{(k)} = -\frac{1}{\langle\mu_k\rangle}\left(\langle e_{nn}^{(k)}\rangle \alpha_{,i}^{(k)} + 2\langle e_{jp}^{(k)}\rangle \beta_{,p}^{(k)}\right) \tag{1.2}$$

$$\varkappa_k = \frac{\langle\lambda_k\rangle + \langle\mu_k\rangle}{\langle\mu_k\rangle}, \quad \lambda_k(\mathbf{x}) = \langle\lambda_k(\mathbf{x})\rangle + \alpha_k(\mathbf{x})$$

$$\mu_k(\mathbf{x}) = \langle\mu_k(\mathbf{x})\rangle + \beta_k(\mathbf{x}) \quad (i, j, m, n, p = 1, 2, 3;\ k = 1, 2)$$

Here and henceforth the summation over the index k is not carried out; $\langle\lambda_k\rangle$ and $\langle\mu_k\rangle$ are the mean values of the Lamé parameters and α_k, β_k denote their fluctuations.

We shall assume that the half-spaces are under constant stresses "at infinity". Although in order to achieve a contact it is sufficient to assume that the stresses normal to the plane of interaction between the half-spaces are not zero, here we adopt the cubic compression which simplifies the computations somewhat, without distorting the meaning of the problem.

The boundary conditions at the plane $x_3 = 0$ have the form

$$u_3^{(1)} = u_3^{(2)}, \quad \sigma_{33}^{(1)} = \sigma_{33}^{(2)}, \quad \sigma_{n3}^{(k)} = 0 \quad (k, n = 1, 2) \tag{1.3}$$

Let us represent the fluctuations in the elastic moduli by the following Fourier integrals ($f_k(\omega)$ and $g_k(\omega)$ are generalized random functions):

$$\{\alpha_k;\ \beta_k\} = \iiint_{-\infty}^{\infty} \{f_k(\omega);\ g_k(\omega)\} \exp(i\omega\mathbf{x})\, d\omega \tag{1.4}$$

$$\mathbf{x} = \{x_1;\ x_2;\ x_3\}, \quad \omega = \{\omega_1;\ \omega_2;\ \omega_3\} \quad (k = 1, 2)$$

We seek a solution of (1.2) in the form

$$v_j^{(k)} = v_{(\mathrm{par})j}^{(k)} + v_{(\mathrm{gen})j}^{(k)} \quad (j = 1, 2, 3;\ k = 1, 2) \tag{1.5}$$

where $v_{(\mathrm{par})j}^{(k)}$ denote the particular solutions of (1.2) and $v_{(\mathrm{gen})j}^{(k)}$ are the general solutions of the corresponding homogeneous systems of equations. Let us put

$$v_{(\mathrm{par})\,j}^{(k)} = \iiint_{-\infty}^{\infty} \gamma_j^{(k)}(\omega) \exp(i\omega\mathbf{x})\, d\omega \tag{1.6}$$

Substituting (1.6) into (1.2) we obtain for $\gamma_j^{(k)}$, as in [2], the following expressions:

$$\gamma_j^{(k)} = i\Bigg[\frac{\langle e_{nn}^{(k)}\rangle}{\langle\mu_k\rangle(1+\varkappa_k)}\frac{\omega_j}{\omega^2} f_k(\omega) + 2\,\frac{(1+\varkappa_k)\,\omega^2\langle e_{jn}^{(k)}\rangle\,\omega_n - \varkappa_k\omega_j\langle e_{pq}^{(k)}\rangle\,\omega_p\omega_q}{\langle\mu_k\rangle(1+\varkappa_k)\,\omega^4}\, g_k(\omega)\Bigg] \tag{1.7}$$

$$\omega^2 = \omega_m\omega_m \quad (j, m, n, p, q = 1, 2, 3;\ k = 1, 2)$$

To find the general solutions of the homogeneous systems corresponding to (1.2), we use the Trefftz's representation [5] which gives a general solution of the equations of the theory of elasticity in terms of harmonic functions

$$v_{(\mathrm{gen})j}^{(k)} = \varphi_{j,3}^{(k)} - a_k x_3 \varphi_{m,jm}^{(k)}$$

$$\varphi_{m,nn}^{(k)} = 0, \quad a_k = \frac{\langle\lambda_k\rangle + \langle\mu_k\rangle}{\langle\lambda_k\rangle + 3\langle\mu_k\rangle} \tag{1.8}$$

$$(j, m, n = 1, 2, 3;\ k = 1, 2)$$

Let us define $\varphi_j^{(k)}$ as follows:

$$\varphi_j^{(k)} = \iint_{-\infty}^{\infty} A_j^{(k)}(\omega_*) \exp \Omega_k\, d\omega_*, \quad \Omega_k = i\omega_*\mathbf{x}_* + (-1)^k\omega_* x_3, \quad \mathbf{x}_* = \{x_1;\ x_2\}, \quad \omega_* = \{\omega_1;\ \omega_2\}, \quad \omega_*^2 = \omega_n\omega_n \quad (k, n = 1, 2) \tag{1.9}$$

Using (1.8) we obtain

$$v_{(\mathrm{gen})\,j}^{(k)} = \iint_{-\infty}^{\infty} [(-1)^k\omega_* A_j^{(k)} + a_k x_3\omega_j Q_A^{(k)}] \exp \Omega_k\, d\omega_* \tag{1.10}$$

$$v^{(k)}_{(\mathrm{gen})3} = (-1)^k \iint\limits_{-\infty}^{\infty} [\omega_* A_3^{(k)} - i a_k x_3 \omega_* Q_A^{(k)}] \exp \Omega_k d\omega_*$$

$$Q_A^{(k)} = \omega_m A_m^{(k)} - (-1)^k i\omega_* A_3^{(k)} \qquad (j,\ k,\ m = 1{,}2)$$

$A_j^{(k)}$ $(j = 1, 2, 3)$ appearing in (1.10) and containing integrals in ω_3 can be found from the boundary conditions (1.3); since the expressions for $A_j^{(k)}$ are bulky, they are not given here.

Substituting (1.6) and (1.10) into (1.5), we obtain the expressions for the displacements in the first order approximation

$$v_j^{(k)} = \iiint\limits_{-\infty}^{\infty} \{\gamma_j^{(k)} \exp(i\boldsymbol{\omega}\mathbf{x}) + [(-1)^k \omega_* B_j^{(k)} + a_k x_3 \omega_j Q_B^{(k)}] \exp \Omega_k d\boldsymbol{\omega} \tag{1.11}$$

$$v_3^{(k)} = \iiint\limits_{-\infty}^{\infty} \{\gamma_3^{(k)} \exp(i\boldsymbol{\omega}\mathbf{x}) + (-1)^k [\omega_* B_3^{(k)} - i a_k x_3 \omega_* Q_B^{(k)}] \exp \Omega_k d\boldsymbol{\omega}$$

$$Q_B^{(k)} = \omega_m B_m^{(k)} - (-1)^k i\omega_* B_3^{(k)}, \quad A_n^{(k)} = \int\limits_{-\infty}^{\infty} B_n^{(k)} d\omega_3 \quad (j,\ k,\ m = 1, 2;$$

$$n = 1,\ 2,\ 3)$$

Using the Cauchy formulas, the Hooke's law

$$\tau_{ij}^{(k)} = 2\left(\langle\mu_k\rangle\, \varepsilon_{ij}^{(k)} + \beta_k \langle e_{ij}^{(k)}\rangle\right) + \left(\langle\lambda_k\rangle\, \varepsilon_{nn}^{(k)} + \alpha_k < \langle e_{nn}^{(k)}\rangle\right) \delta_{ij}$$

and the relations (1.11), we can obtain the expressions for the deformation and stress perturbations. In what follows, we shall limit ourselves to the deformations.

The expressions describing the fluctuations in deformations have the following form:

$$\varepsilon_{mn}^{(k)} = \frac{i}{2} \iiint\limits_{-\infty}^{\infty} \{(\gamma_m^{(k)} \omega_n + \gamma_n^{(k)} \omega_m) \exp(i\boldsymbol{\omega}\mathbf{x}) + \tag{1.12}$$

$$[(-1)^k \omega_* (\omega_n B_m^{(k)} + \omega_m B_n^{(k)}) + 2 a_k \omega_m \omega_n x_3 Q_B^{(k)}] \exp \Omega_k\} d\boldsymbol{\omega}$$

$$\varepsilon_{m3}^{(k)} = \frac{1}{2} \iiint\limits_{-\infty}^{\infty} \{i(\gamma_m^{(k)} \omega_3 + \gamma_3^{(k)} \omega_m) \exp(i\boldsymbol{\omega}\mathbf{x}) + [(-1)^k \omega_* ((-1)^k \omega_* B_m^{(k)} +$$

$$i\omega_m B_3^{(k)}) + a_k \omega_m (1 + (-1)^k 2\omega_* x_3)\, Q_B^{(k)}] \exp \Omega_k\} d\boldsymbol{\omega}$$

$$\varepsilon_{33}^{(k)} = \iiint\limits_{-\infty}^{\infty} \{i\gamma_3^{(k)} \omega_3 \exp(i\boldsymbol{\omega}\mathbf{x}) + (-1)^k [(-1)^k B_3^{(k)} -$$

$$i a_k (1 + (-1)^k \omega_* x_3)\, Q_B^{(k)}] \exp \Omega_k\} d\boldsymbol{\omega}$$

$$\theta_k = v_{i,i}^{(k)} = i \iiint\limits_{-\infty}^{\infty} [\gamma_p^{(k)} \omega_p \exp(i\boldsymbol{\omega}\mathbf{x}) + (-1)^k (1 - a_k) \times$$

$$\omega_* Q_B^{(k)} \exp \Omega_k] d\boldsymbol{\omega} \quad (k,\ m,\ n = 1,\ 2;\ p = 1,\ 2,\ 3)$$

The formulas for the displacements and deformations in the half-spaces $x_3 \geqslant 0$ and $x_3 \leqslant 0$ contain Fourier transforms $f_k(\boldsymbol{\omega})$ and $g_k(\boldsymbol{\omega})$ of the functions α_k and

β_k $(k = 1, 2)$, and this would seem to imply that in order to determine the deformations in e.g. the half-space $x_3 \geqslant 0$, the functions must be defined over the whole space. It can be shown that this is not true, i.e. that the values λ_1 and μ_1 assumed for $x_3 < 0$ and λ_2, μ_2 for $x_3 > 0$, , do not affect the values of the deformations for $x_3 \geqslant 0$.

We shall carry out all computations for the case $\varepsilon_{13}^{(1)}$. The computations for the remaining deformations and for the stresses are identical.

Let us write the expressions for f_1 and g_1 in the form

$$\{f_1; g_1\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \{F; G\} \exp(-i\omega_3 u_3)\, du_3$$

$$F = F(\omega_1; \omega_2; u_3), \quad G = G(\omega_1; \omega_2; u_3)$$

Using these expressions we can reduce the Fourier transformation of $\varepsilon_{13}^{(1)}$ in x_1 and x_2 to the form

$$P_{13}(\omega_1; \omega_2; x_3) = \int_{-\infty}^{\infty} R_{13}(\omega_1; \omega_2; x_3; u_3)\, du_3 \tag{1.13}$$

$$R_{13} = \frac{1}{2\pi} T_{13}(\omega_1; \omega_2; u_3) \int_{-\infty}^{\infty} \Big\{ -\omega_3 \exp(i\omega_3 x_3) +$$
$$\left[\omega_3 - \frac{8\varkappa_1(1+\varkappa_2)\langle\mu_1\rangle}{(2+\varkappa_1)(2+\varkappa_2)D}(\omega_3 - i\omega_*)\right] \times$$
$$\exp(-\omega_* x_3)\Big\} \frac{\exp(i\omega_3 u_3)}{\omega^2}\, d\omega_3$$

$$D = \frac{16}{(2+\varkappa_1)(2+\varkappa_2)}[\varkappa_1(1+\varkappa_2)\langle\mu_1\rangle + \varkappa_2(1+\varkappa_1)\langle\mu_2\rangle]$$

Since the functions f_k, g_k $(k = 1, 2)$ are independent, only $f_1 \neq 0$ appears in (1.13) and $f_2 = g_1 = g_2 = 0$. Direct computation of the integral in (1.13) yields $\varepsilon_{13}^{(1)} = 0$ and this shows that the deformations in the half-space $x_3 \geqslant 0$ are independent of the values assumed by the moduli of elasticity at $x_3 < 0$ (the computations for the case $g_1 \neq 0$, $f_1 = f_2 = g_2 = 0$ are analogous and are not given here).

Next we shall show that the deformations in $x_3 \geqslant 0$ are independent of the values assumed by the moduli λ_2 and μ_2 in $x_3 > 0$. Let us put $f_2 \neq 0$, $f_1 = g_1 = g_2 = 0$. Then

$$R_{13} = \frac{1}{2\pi} T_{13} \int_{-\infty}^{\infty} (\omega_3 + i\omega_*) \exp(-\omega_* x_3) \frac{\exp(-i\omega_3 u_3)}{\omega^2}\, d\omega_3 = 0$$

which proves the above assertion.

2. Let us inspect the deformed state of the half-space $x_3 \geqslant 0$ in more detail. All arguments which follow also hold for the half-space $x_3 \leqslant 0$.

Let the random fields α_1 and β_1 be statistically homogeneous and isotropic, related to each other in a statistically homogeneous and isotropic manner, and have the known correlation functions

$$K_{yz}(\xi) = \langle \overline{y_1(\mathbf{x})}\, z_1(\mathbf{x}+\xi)\rangle \quad (y, z = \alpha, \beta) \tag{2.1}$$

Here and henceforth the bar denotes a complex conjugate quantity.

We make the same assumptions about α_2 and β_2 as about α_1 and β_1, and we also assume that

$$\langle \bar{y}_1 z_2 \rangle = 0 \quad (y, z = \alpha, \beta) \tag{2.2}$$

and this leads to the following expressions for the components of the correlational deformation tensor:

$$K_{pqst}(\mathbf{x}, \xi) = \langle \overline{\varepsilon_{pq}^{(1)}(\mathbf{x})}\, \varepsilon_{st}^{(1)}(\mathbf{x}+\xi)\rangle \tag{2.3}$$

From (2.3), (1.12), (2.1) and (2.2) we see that the deformation field is stationary in the directions of the x_1 and x_2 axes, and nonstationary in the direction of the x_3-axis. Without quoting the bulky expressions for K_{pqst} for all values of p, q, s and t, we shall consider the correlation functions of the deformation $\varepsilon_{12}^{(1)}$ and volume expansion θ_1.

The behavior of the correlation functions and the dispersion of deformations at the boundary of interaction between the half-space is of particular interest. From (1.12) we have, for $x_3 = 0$

$$\varepsilon_{12}^{(1)} = \frac{i}{2} \iiint_{-\infty}^{\infty} [\gamma_1\omega_2 + \gamma_2\omega_1 - \omega_* (\omega_2 B_1 + \omega_1 B_2)] \exp(i\omega_* x_*) d\omega \tag{2.4}$$

$$\theta_1 = i \iiint_{-\infty}^{\infty} [\gamma_m\omega_m + (a_1 - 1)(\omega_n B_n + i\omega_* B_3)] \exp(i\omega_* x_3)\, d\omega$$

$$(m = 1, 2, 3;\ n = 1, 2)$$

Using (2.4), (2.2) and (2.3), we obtain the correlation functions for $\varepsilon_{12}^{(1)}$ and θ_1

$$K_{1212}(\xi_*) = \iiint_{-\infty}^{\infty} \Big\{ b_4{}^2 \frac{\omega_1{}^2\omega_2{}^2}{\omega^4} S_1(\omega) + b_3{}^2 \frac{\omega_1{}^2\omega_2{}^2}{\omega_*{}^2} \Big[\frac{\omega_2{}^2\omega_3{}^2}{\omega_*{}^4} b_k S_k(\omega) + (1+\varkappa_2) c_k S_k(\omega) \Big] - 2(1+\varkappa_2) b_3 b_4 \Big(d_1 + 2h_1 \frac{\omega_3{}^2}{\omega^2}\Big) \frac{\omega_1{}^2\omega_2{}^2}{\omega_*{}^2\omega^2} \times S_1(\omega) \Big\} \exp(i\omega_* x_*)\, d\omega \tag{2.5}$$

$$K_\theta(\xi_*) = \iiint_{-\infty}^{\infty} \Big[h_1 \Big(\frac{1}{\langle\mu_1\rangle^2} + d_3 \Big(d_3 + \frac{2}{\langle\mu_1\rangle}\Big) \frac{\omega_*{}^2}{\omega^2} \Big) S_1(\omega) + d_3{}^2 h_2{}^2 \frac{\omega_*{}^4}{\omega^4} S_2(\omega) \Big] \exp(i\omega_* x_*)\, d\omega$$

$$b_1 = 4\langle e^{(1)}\rangle \Big[1 + \varkappa_2 \Big(1 + \frac{\langle\mu_2\rangle}{\langle\mu_1\rangle} \Big) \Big], \quad b_2 = 4\langle e^{(2)}\rangle \frac{1-\varkappa_2}{1+\varkappa_2}$$

$$b_3 = \frac{2}{(2+\varkappa_1)(2+\varkappa_2) D}, \quad b_4 = \frac{\langle e^{(1)}\rangle}{(1+\varkappa_1)\langle\mu_1\rangle}$$

$$c_k = \Big(d_k - 2h_k \frac{\omega_3{}^2}{\omega^2} \Big), \quad d_k = (-1)^k \langle e^{(k)}\rangle \Big(1 - \frac{\langle\lambda_k\rangle}{(1+\varkappa_k)\langle\mu_k\rangle} \Big)$$

$$h_k = \langle e^{(k)} \rangle \frac{1}{1+\varkappa_k}, \quad d_3 = 8(1+\varkappa_2) b_3$$
$$S_k = 9\, S_{11}{}^{(k)} + 12\, S_{12}{}^{(k)} + 4\, S_{22}{}^{(k)}, \quad \xi_*{}^2 = \xi_k \xi_k$$
$$\langle e_{ij}{}^{(k)} \rangle = \langle e^{(k)} \rangle \delta_{ij} \quad (k = 1,\, 2)$$

where $S_{ij}{}^{(k)}$ denote the spectral densities of the fields α_k and β_k.

Passing in (2.5) to spherical coordinates and integrating over the angles, we obtain ($J_{m/n}$ is a Bessel function)

$$K_{1212}(\xi_*) = \frac{\pi^2}{4} \int_0^\infty \{b_4{}^2 H_1 S_1(\omega) + b_3{}^2 [(H_2 - H_1) b_k{}^2 S_k(\omega) + \tag{2.6}$$
$$(1+\varkappa_2)^2 c_k' S_k(\omega)] - 2(1+\varkappa_2) b_3 b_4 [d_1 H_2 + 2h_1 (H_2 - H_1)] S_1(\omega)\} \omega^2 d\omega$$

$$K_\theta(\xi_*) = 2\pi^2 \int_0^\infty \left\{h_1{}^2 \left[\frac{1}{\langle\mu_1\rangle^2} + d_3\left(d_3 + \frac{1}{\langle\mu_1\rangle}\right) \frac{1}{\omega^2} (-\nabla^2) S_1(\omega) + d_3{}^2 h_2{}^2 \frac{1}{\omega^4} \nabla^4 S_2(\omega)\right\} J_{1/2}(\eta) J_{-1/2}(\eta)\, \omega^2 d\omega\right.$$

$$H_1 = \frac{1}{16} [J_{5/2}(\eta) J_{-5/2}(\eta) + 5 J_{3/2}(\eta) J_{-3/2}(\eta) + 10 J_{1/2}(\eta) J_{-1/2}(\eta) - (J_{5/2}(\eta) J_{-1/2}(\eta) + 5 J_{7/2}(\eta) J_{1/2}(\eta) + 10 J_{3/2}(\eta) J_{3/2}(\eta)) \cos\psi]$$

$$H_2 = \frac{1}{4} [J_{3/2}(\eta) J_{-3/2}(\eta) + 3 J_{1/2}(\eta) J_{-1/2}(\eta) - (J_{5/2}(\eta) J_{1/2}(\eta) + 3 J_{3/2}(\eta) J_{3/2}(\eta)) \cos\psi]$$

$$H_3 = J_{1/2}(\eta) J_{-1/2}(\eta) - J_{5/2}(\eta) J_{3/2}(\eta) \cos\psi$$

$$\eta = \frac{\omega \xi_*}{2}, \quad \psi = \operatorname{arctg} \frac{\xi_1}{\xi_2}, \quad \nabla^2 = \frac{\partial^2}{\partial \xi_1{}^2} + \frac{\partial^2}{\partial \xi_2{}^2}$$

$$c_m' = d_m{}^2 H_3 - 4\, d_m h_m (H_3 - H_2) + 4\, h_m{}^2 (H_3 - 2\, H_2 + H_1)$$

($k, m = 1, 2,$ in m do not summarize)

Setting in (2.6) $\xi_* = 0$, we obtain the expressions for dispersions

$$D_{1212} = \frac{\pi}{30} \int_0^\infty \{8 b_4{}^2 S_1(\omega) + b_3{}^2 [2 b_k{}^2 S_k(\omega) + (1+\varkappa_2)^2 c_k' S_k(\omega)] - \tag{2.7}$$
$$4(1+\varkappa_2) b_3 b_4 (5 d_1 + 2 h_1) S_1(\omega)\} \omega^2 d\omega$$

$$D_\theta = 2\pi \int_0^\infty \left\{h_1{}^2 \left[\frac{2}{\langle\mu_1\rangle^2} + \frac{4}{3} d_3 \left(d_3 + \frac{2}{\langle\mu_1\rangle}\right)\right] S_1(\omega) + \frac{16}{15} d_3{}^2 h_2{}^2 S_2(\omega)\right\} \omega^2 d\omega \quad (k = 1,\, 2)$$

3. Let us consider some limiting cases.

1°. The half-spaces are filled with the same medium, i.e. $\lambda_1 = \lambda_2$ and $\mu_1 = \mu_2$. In this case the expressions (2.7) will become

$$D_{1212} = \frac{4}{15} r^2 D^* v_{1212}^2, \quad D_\theta = 2 r^2 D^* v_\theta{}^2$$

$$D^* = \int_0^\infty \omega^2 S(\omega)\, d\omega, \quad r = \frac{\pi \langle e \rangle}{(1+\varkappa)\langle \mu_1 \rangle}$$

Here v_{1212} and v_θ are the variability coefficients. Computing these coefficients yields the following results: $v_{1212}^2 = 0{,}10$, $v_\theta^2 = 1.96$ (the Poisson's ratio $\nu = 0.25$), and their values for the whole space [4] are $v_{1212}^2 = 0.093$, $v_\theta^2 = 1.39$.

2°. The half-space $x_3 \leqslant 0$ is homogeneous. Assuming that the mean values of the elastic moduli for $x_3 \geqslant 0$ are equal to the values of the elastic moduli of the half-space $x_3 \leqslant 0$, we obtain $v_{1212}^2 = 0.25$, $v_\theta^2 = 2.38$.

3°. The half-space $x_3 \leqslant 0$ is perfectly rigid. In this case the dispersion of the volume expansion at the boundary $x_3 = 0$ coincides with the dispersion of the volume expansion over the whole space, and the variability coefficient $v_{1212}^2 = 0.116$.

4°. The half-space $x_3 \geqslant 0$ is loaded along the plane $x_3 = 0$ by normal stresses. We have $v_{1212}^2 = 0.198$, $v_\theta^2 = 2.55$.

Using (1.12) and the Hooke's law we can obtain the expressions for the stress fluctuations and hence the formulas for the components of the correlation and stress dispersion tensors. Without quoting these formulas in full, we shall give just the values of the variability coefficient for the stresses $\sigma_{11}^{(1)}$ (or $\sigma_{22}^{(1)}$) and $\sigma_{12}^{(1)}$ at the boundary of interaction between the half-spaces. For the cases 1° — 4° we have, respectively, $v_{1111}^2 = v_{2222}^2 = 0.431$, 0.101, 0.107, 0.116; $v_{1212}^2 = 0.1$, 0.25, 0.116, 0.13, and we have $v_{1111}^2 = v_\theta^2 = 0.105$, $v_{1212}^2 = 0.093$ in the whole space [4].

Comparison of the values of the variability coefficients obtained here with the corresponding values given in [6], is of interest. Let us compare the ratio of the variability coefficients for the stresses $\sigma_{11}^{(1)}$ ($\sigma_{22}^{(1)}$), $\sigma_{12}^{(1)}$ in the half-space to the corresponding coefficients in the whole space (the values of w from [6] are given in brackets) $w_{1111} = 1.104$ (1.099), $w_{1212} = 2.123$ (2.109). We see that the corresponding values differ from each other by fractions of one percent. It follows that we observe a considerable concentration of the deformations (stresses) at the boundary between the half-spaces, which should be allowed for in practical computations.

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